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MOTIVE FORCE IN A COMPLEX MODEL OF MASS TRANSFER

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Relations for determining a motive force in a complex model of mass transfer as applied to concurrent, countercurrent, and cross motion of the vapor and liquid phases have been introduced. These forces have been compared for different forms of organization of the flow, and their comparison with similar quantities from the known Murphree and Hausen models is presented.

The complex model of mass transfer whose main principles for concurrent, countercurrent, and cross motion of vapor and liquid are given in [1–3] generalizes the known Murphree and Hausen models [4, 5] and presents them as boundary cases.

In [6], it is noted that motive forces and the numbers of transfer units are individual for each model and form of organization of motion of the phases but in a generalized form they are expressed by known relations. In this case, the conditions of the relation between ideal and real plates which are typical of the Murphree and Hausen models and hold for complete mixing of vapor and liquid or concurrent flow are extended to countercurrent and cross motion of flows. We consider special features of the expression for a motive force in the complex model for more familiar schemes of organization of flows (see Fig. 1).

In [7], it is suggested to determine the distance from the place of introduction of phases to the surfaces, where concentrations in the ideal and real plates are leveled, as a function of the coefficient of phase equilibrium. With account for this special feature of the complex model, which imparts a dynamic character to it and allows correction of the form of the computational formulas with changes in the coefficient of phase equilibrium, from [1–3] we obtained concentrations of an easily volatile component in the liquid entering the real plate and the vapor leaving it, respectively:

for concurrent motion

$$x_n = x_{n-1} + \frac{(m+1)\left(x_{n-1} - \frac{y_{n-1}}{m}\right)E_{\mathrm{con}}}{\frac{L}{V} + m + \frac{L}{mV}E_{\mathrm{con}} - mE_{\mathrm{con}}}\,, \tag{1}$$

$$y_n = y_{n-1} + \frac{(m+1)\left(x_{n-1} - \frac{y_{n-1}}{m}\right)\frac{L}{V}E_{\mathrm{con}}}{\frac{L}{V} + m + \frac{L}{mV}E_{\mathrm{con}} - mE_{\mathrm{con}}}\,; \tag{2}$$

for countercurrent motion

$$x_n = x_{n-1} + \frac{(m+1)\left(x_{n-1} - \frac{y_{n-1}}{m}\right)E_{\mathrm{g}}}{\frac{L}{V} - 1 + \frac{L}{mV}E_{\mathrm{g}} - mE_{\mathrm{g}}}\,, \tag{3}$$

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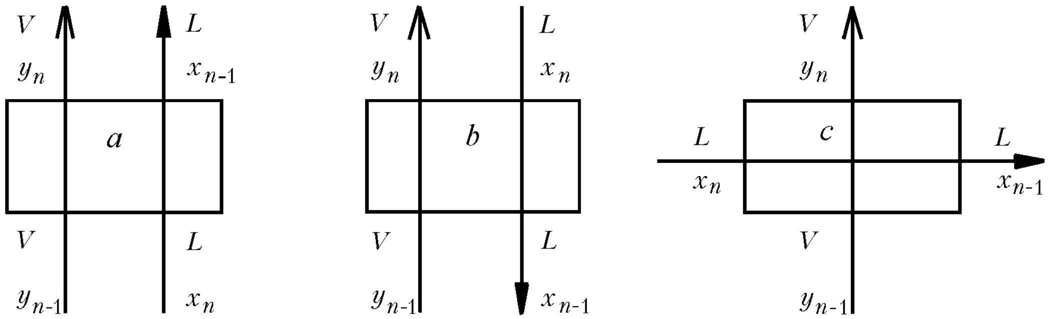

Fig. 1. Schemes of organization of vapor and liquid flows: a) concurrent motion; b) countercurrent motion; c) cross motion.

$$y_n = y_{n-1} + \frac{(m+1)\left(x_{n-1} - \frac{y_{n-1}}{m}\right)\frac{L}{V}E_{\text{g}}}{\frac{L}{V} - 1 + \frac{L}{mV}E_{\text{g}} - mE_{\text{g}}}; \tag{4}$$

for cross motion

$$x_n = x_{n-1} + \frac{(m+1)\left(x_{n-1} - \frac{y_{n-1}}{m}\right)E_{\text{k}}}{\frac{L}{V} + \frac{m-1}{2} + \frac{L}{mV}E_{\text{k}} - mE_{\text{k}}}, \tag{5}$$

$$y_n = y_{n-1} + \frac{(m+1)\left(x_{n-1} - \frac{y_{n-1}}{m}\right)\frac{L}{V}E_{\text{k}}}{\frac{L}{V} + \frac{m-1}{2} + \frac{L}{mV}E_{\text{k}} - mE_{\text{k}}}. \tag{6}$$

Substituting (1)–(6) into the known equations of the average motive forces, we obtain the formulas expressed in terms of the parameters of the liquid and vapor flows:

in concurrent motion

$$\Delta x_{\text{mean,log}} = \frac{\left(\frac{L}{mV} + 1\right)(x_n - x_{n-1})}{\ln \frac{1 + \frac{E_{\text{con}}}{m}}{1 - E_{\text{con}}}}, \tag{7}$$

$$\Delta y_{\text{mean,log}} = \frac{\left(1 + \frac{mV}{L}\right)(y_n - y_{n-1})}{\ln \frac{1 + \frac{E_{\text{con}}}{m}}{1 - E_{\text{con}}}}; \tag{8}$$

in countercurrent motion

TABLE 1. Arithmetic Mean Forces in Concurrent and Cross Motion

Quantities	Concurrent motion	Cross motion
$\dfrac{\Delta x_{\text{mean,a}}}{x_n - x_{n-1}}$	$\dfrac{\dfrac{L}{V} + m - \dfrac{E_{\text{con}}}{2}\left(\dfrac{L}{mV} + 1\right)(m-1)}{(m+1)E_{\text{con}}}$	$\dfrac{\dfrac{L}{V} + \dfrac{m-1}{2} - \dfrac{E_{\text{k}}}{2}\left(\dfrac{L}{mV} + 1\right)(m-1)}{(m+1)E_{\text{k}}}$
$\dfrac{\Delta y_{\text{mean,a}}}{y_n - y_{n-1}}$	$\dfrac{m + \dfrac{m^2 V}{L} - \dfrac{E_{\text{con}}}{2}\left(1 + \dfrac{mV}{L}\right)(m-1)}{(m+1)E_{\text{con}}}$	$\dfrac{m + \dfrac{mV}{L}\dfrac{m-1}{2} - \dfrac{E_{\text{k}}}{2}\left(1 + \dfrac{mV}{L}\right)(m-1)}{(m+1)E_{\text{k}}}$

$$\Delta x_{\text{mean,log}} = \frac{\left(\dfrac{L}{mV} - 1\right)(x_n - x_{n-1})}{\ln \dfrac{1 + \dfrac{\dfrac{L}{mV} - m}{\dfrac{L}{V} - 1} E_{\text{g}}}{1 - E_{\text{g}}}}, \tag{9}$$

$$\Delta y_{\text{mean,log}} = \frac{\left(1 - \dfrac{mV}{L}\right)(y_n - y_{n-1})}{\ln \dfrac{1 + \dfrac{\dfrac{L}{mV} - m}{\dfrac{L}{V} - 1} E_{\text{g}}}{1 - E_{\text{g}}}}; \tag{10}$$

in cross motion

$$\Delta x_{\text{mean,log}} = \frac{\dfrac{L}{mV}(x_n - x_{n-1})}{\ln \dfrac{1 + \dfrac{\dfrac{L}{mV} - \dfrac{m-1}{2}}{\dfrac{L}{V} + \dfrac{m-1}{2}} E_{\text{k}}}{1 - E_{\text{k}}}}, \tag{11}$$

$$\Delta y_{\text{mean,log}} = \frac{\dfrac{L}{mV} y_n - y_{n-1}}{\ln \dfrac{1 + \dfrac{\dfrac{L}{mV} - \dfrac{m-1}{2}}{\dfrac{L}{V} + \dfrac{m-1}{2}} E_{\text{k}}}{1 - E_{\text{k}}}}. \tag{12}$$

It is seen from formulas (9) and (10) that in countercurrent motion and with parallelicity of the working line and the equilibrium line of the current section the determination of the logarithmic mean of the motive force is difficult, since L/V = 1 and, in this case, it is worth finding it as an arithmetical mean

$$\Delta x_{\text{mean,a}} = \frac{\left[\frac{L}{V} - 1 - \frac{E_{\text{g}}}{2}\left(\frac{L}{mV} + 1\right)(m-1)\right]}{(m+1)\, E_{\text{g}}} (x_n - x_{n-1}) \,, \tag{13}$$

$$\Delta y_{\text{mean,a}} = \frac{m - \frac{mV}{L} - \frac{E_{\text{g}}}{2}\left(1 + \frac{mV}{L}\right)(m-1)}{(m+1)\, E_{\text{g}}} (y_n - y_{n-1}) \,. \tag{14}$$

Similar quantities are also obtained for concurrent and cross motion (Table 1).

It follows from the analysis of formulas (7)–(14) that the mean motive force at the same efficiency and $m > 1$ is maximum for concurrent motion and minimum for countercurrent motion, since the change in the motive forces is mostly determined by the numerator of the complete fraction. The numerators of the logarithm number change similarly but their effect is less substantial.

It follows from comparison with the data of [6] that the motive force in concurrent motion in the complex model is smaller than in the Hausen model but larger than in the Murphree model: in analysis of the efficiency in the vapor phase for $L(m^2V) < 1$ and in the liquid for $L/V > 1$. In countercurrent motion, the motive force in the complex model is smaller than in the models where the conditions of relation between the ideal and real plates of the Hausen model, the Murphree model in the analysis of the efficiency in the vapor phase, and the hypothetical model, for which $L/(mV) > m$. In cross motion of flows, the motive force is also smaller than in the model which uses the conditions of relation between the ideal and real plates of the Hausen model and the hypothetical model and is larger than in the model which uses the conditions of relation between the ideal and real plates of the Murphree model in the analysis of the efficiency in the vapor and liquid phases at a certain ratio of the parameters.

When the kinetics is determined by high values of the transfer units, in addition to the differences of concentrations the motive force can be expressed by the numbers of transfer units N which are related to the differences of concentrations by the known relations [6, 8] for the liquid and vapor phases:

$$N_{\text{liq}} = \frac{x_n - x_{n-1}}{\Delta x_{\text{mean}}} \,, \tag{15}$$

$$N_{\text{v}} = \frac{y_n - y_{n-1}}{\Delta y_{\text{mean}}} \,. \tag{16}$$

In particular, for concurrent motion, formulas (15) and (16) with account for (7) and (8) take on the form, respectively,

$$N_{\text{con,liq,log}} = \frac{\ln \dfrac{1 + \frac{E_{\text{con}}}{m}}{1 - E_{\text{con}}}}{\frac{L}{mV} + 1} \,, \tag{17}$$

$$N_{\text{con,v,log}} = \frac{\ln \dfrac{1 + \frac{E_{\text{con}}}{m}}{1 - E_{\text{con}}}}{1 + \frac{mV}{L}} \,. \tag{18}$$

Such dependences can easily be found for countercurrent and cross motion by a combined solution of (15), (16), and (9)–(12).

TABLE 2. Motive Forces for Mixtures Approaching Ideal Ones

Quantities	Concurrent motion	Countercurrent motion	Cross motion
$\Delta x_{\text{mean,log}}$	$\dfrac{\left(\frac{L}{mV}+1\right)(x_n-x_{n-1})}{\ln\frac{1+E_{\text{con,m}}}{1-E_{\text{con,m}}}}$	$\dfrac{\left(\frac{L}{mV}-1\right)(x_n-x_{n-1})}{\ln\frac{1+E_{\text{g,m}}}{1-E_{\text{g,m}}}}$	$\dfrac{\frac{L}{mV}(x_n-x_{n-1})}{\ln\frac{1+E_{\text{k,m}}}{1-E_{\text{k,m}}}}$
$\Delta y_{\text{mean,log}}$	$\dfrac{\left(1+\frac{mV}{L}\right)(y_n-y_{n-1})}{\ln\frac{1+E_{\text{con,m}}}{1-E_{\text{con,m}}}}$	$\dfrac{\left(1-\frac{mV}{L}\right)(y_n-y_{n-1})}{\ln\frac{1+E_{\text{g,m}}}{1-E_{\text{g,m}}}}$	$\dfrac{y_n-y_{n-1}}{\ln\frac{1+E_{\text{k,m}}}{1-E_{\text{k,m}}}}$
$\Delta x_{\text{mean,a}}$	$\dfrac{\left(\frac{L}{mV}+1\right)(x_n-x_{n-1})}{2E_{\text{con,m}}}$	$\dfrac{\left(\frac{L}{mV}-1\right)(x_n-x_{n-1})}{2E_{\text{g,m}}}$	$\dfrac{\frac{L}{mV}(x_n-x_{n-1})}{2E_{\text{k,m}}}$
$\Delta y_{\text{mean,a}}$	$\dfrac{\left(1+\frac{mV}{L}\right)(y_n-y_{n-1})}{2E_{\text{con,m}}}$	$\dfrac{\left(1-\frac{mV}{L}\right)(y_n-y_{n-1})}{2E_{\text{g,m}}}$	$\dfrac{y_n-y_{n-1}}{2E_{\text{k,m}}}$
$N_{\text{liq,log}}$	$\dfrac{\ln\frac{1+E_{\text{con,m}}}{1-E_{\text{con,m}}}}{\frac{L}{mV}+1}$	$\dfrac{\ln\frac{1+E_{\text{g,m}}}{1-E_{\text{g,m}}}}{\frac{L}{mV}-1}$	$\dfrac{mV}{L}\ln\dfrac{1+E_{\text{k,m}}}{1-E_{\text{k,m}}}$
$N_{\text{v,log}}$	$\dfrac{\ln\frac{1+E_{\text{con,m}}}{1-E_{\text{con,m}}}}{1+\frac{mV}{L}}$	$\dfrac{\ln\frac{1+E_{\text{g,m}}}{1-E_{\text{g,m}}}}{1-\frac{mV}{L}}$	$\ln\dfrac{1+E_{\text{k,m}}}{1-E_{\text{k,m}}}$
$N_{\text{liq,a}}$	$\dfrac{2E_{\text{con,m}}}{\frac{L}{mV}+1}$	$\dfrac{2E_{\text{g,m}}}{\frac{L}{mV}-1}$	$2\dfrac{mV}{L}E_{\text{k,m}}$
$N_{\text{v,a}}$	$\dfrac{2E_{\text{con,m}}}{1+\frac{mV}{L}}$	$\dfrac{2E_{\text{g,m}}}{1-\frac{mV}{L}}$	$2E_{\text{k,m}}$

The complex model also assumes the version of separation of an ideal mixture for which the coefficient of phase equilibrium is equal to unity. In this case, the distances h and h_1 are half the total value ($h = h_1 = 0.5$). The values of the motive forces in separation of an ideal mixture are derived from (7)–(12) at $m = 1$. Separation of such solutions cannot be realized and motive forces are of practical importance when the coefficient of phase equilibrium is not equal to but is close to unity. These situations arise as the concentration of an easily volatile component approaches a maximum value or a value at the azeotropic point from both smaller and larger quantities.

In [1–3], formulas similar to (1), (3), and (5), but for which $h = h_1 = 0.5$, are obtained for concurrent, countercurrent, and cross motion. The same dependences expressed in terms of the parameters of the vapor phase have the form for

concurrent motion

$$y_n = y_{n-1} + \frac{2\left(x_{n-1} - \frac{y_{n-1}}{m}\right)\frac{L}{V}E_{\text{con,m}}}{\frac{L}{mV} + 1 + \frac{L}{mV}E_{\text{con,m}} - mE_{\text{con,m}}}, \tag{19}$$

countermotion

$$y_n = y_{n-1} + \frac{2\left(x_{n-1} - \frac{y_{n-1}}{m}\right)\frac{L}{V} E_{\text{g,m}}}{\frac{L}{mV} - 1 + \frac{L}{mV} E_{\text{g,m}} - mE_{\text{g,m}}}, \tag{20}$$

cross motion

$$y_n = y_{n-1} + \frac{2\left(x_{n-1} - \frac{y_{n-1}}{m}\right)\frac{L}{V} E_{\text{k,m}}}{\frac{L}{mV} + \frac{L}{mV} E_{\text{k,m}} - mE_{\text{k,m}}}. \tag{21}$$

In substitution of formulas (20), (14), and (13) from [1–3], respectively, and (19)–(21) into the known equations for the motive forces, we found their logarithmic and arithmetic mean values, which are expressed in terms of the parameters of the liquid and vapor phases, and the nuclear of transfer units (Table 2).

It follows from the analysis of the dependences obtained that the expressions for different forms of organization of flows have the same denominators but differ in their numerators, and the numbers of transfer units, on the contrary, differ in their denominators. Substitution of $m = 1$ into formulas (7)–(12) and the corresponding expressions (see Table 2) leads, correspondingly, to the same results, which confirms the universal character of the complex model.

NOTATION

E, efficiency of the plate; h and h_1, dimensionless distances from the place of entry of vapor and liquid, respectively, to the surface of the equality of concentrations of phases in the ideal and real plates; L, molar liquid flow; m, coefficient of phase equilibrium; N, number of transfer units; V, molar vapor flow; x and y, concentrations of the easily volatile component in liquid and vapor, respectively. Subscripts: a, arithmetic mean value; g, counterflow motion; k, cross motion; log, logarithmic mean value; liq, liquid; m, values at the coefficient of phase equilibrium equal to unity; n, number of the plate under consideration; $n - 1$, number of the previous plate along vapor motion; con, concurrent motion; mean, mean value; v, vapor phase.

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